The Bernoulli Inverse Method applied to Argo float data in the Atlantic

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What are we trying to do?

The circulation of the Atlantic Ocean is a major component of the climate system of the Earth. It is important then for us to find ways to monitor how it is changing in space and time.

The Argo Programme has released 2410 autonomous floats (as at 27/02/2006) into the world ocean each of which provides a vertical profile of pressure, temperature and salinity every 10 days to a typical depth of 2000m.

To generate a picture of the circulation of the Atlantic, these data could be assimilated into a large and complex model of the atmosphere ocean system. An alternative simpler approach is to use the profiles to calculate horizontal density gradients and therefore the geostrophic velocity relative to some level.

This leaves the problem of knowing the velocity at a particular level. This is where the Bernoulli inverse comes in.

What is the Bernoulli Inverse Method?

The Bernoulli method uses vertical profiles of pressure, temperature and salinity data to make estimates of sea surface height, and therefore from the horizontal gradient of height, the surface velocity.

It does this by using oceanic properties that are conserved along steady geostrophic streamlines.

First each profile is assigned a set of nearest neighbours (10 in the examples here).

Next, if two nearby profiles have two conserved properties (salinity and a modified potential temperature) that have the same values at a point on each, it is assumed that a geostrophic streamline connects them. If this is the case, then any other conserved property must have the same value at the two points as well.

The Bernoulli function is just such a conserved property, and it can be expressed as the sum of a known function of pressure, temperature and salinity, and the unknown sea surface height.

Since the function has the same total value at the two points, the difference in sea surface height between the two points is known.

All such differences can be assembled into a matrix equation of the form:

$$A\eta = b$$

and solved by Singular Value Decomposition (SVD).

Here η is a column vector of n sea surface heights corresponding to n Argo floats; b is a column vector of m Bernoulli differences corresponding to each crossing point found; and A is an n by m matrix each row of which has only the two non-zero elements 1 and -1.

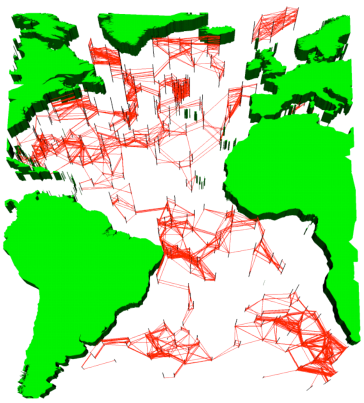

Figure 1. Float profiles (black lines) and connecting lines (red) for Argo floats in the Atlantic at 1st April 2005

The mesh of profiles and connecting pseudo streamlines is shown in Figure 1. These have been calculated using 10 days of float data in April. Figure 2 shows the resulting sea surface height field and error.

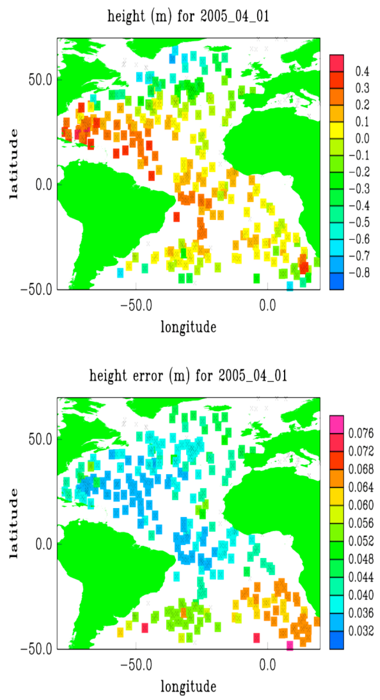

Figure 2. Bernoulli Inverse solution for 1st April 2005 using the previous 10 days of Argo float data. The top panel shows the sea surface height, the bottom panel the associated error, both in meters.

Including Altimetry

Altimeter data can be used by including in the matrix equation the difference between the sea surface height at two different times.

This requires a solution to the equation:

$$\begin{pmatrix} A(t_1) & & & \\ & A(t_2) & & \\ & & -I & I \end{pmatrix}\begin{pmatrix}\eta(t_1)\\ \eta(t_2)\\ h(t_1)\\ h(t_2)\end{pmatrix} = \begin{pmatrix} b(t_1)\\ b(t_2)\\ H(t_2)-H(t_1)\end{pmatrix}$$

Here t_1 and t_2 are two time levels, **I** is the identity matrix, h is the sea surface height at altimeter points and H is the altimeter measurement from these points.

Unfortunately in general float points and altimeter points will not coincide so that the above equation will not yield a useful solution.

To couple the two sets of observation points we introduce a polynomial representation for the sea surface height. For a polynomial of order K, we use terms of the form:

$$\alpha_n x^n y^{K-n} \quad n = 1,..,K$$

Where x and y are the east-west and north-south coordinate and α_n is the n'th unknown coefficient to be determined. The new equation to solve by SVD is then:

$$\begin{pmatrix} A(t_1) & & & \\ & A(t_2) & & \\ & & -I & I \end{pmatrix}\begin{pmatrix} P(t_1) & \\ & P(t_2) \\ Q(t_1) & \\ & Q(t_2) \end{pmatrix}\begin{pmatrix}\alpha(t_1)\\ \alpha(t_2)\end{pmatrix} = \begin{pmatrix} b(t_1)\\ b(t_2)\\ H(t_2)-H(t_1)\end{pmatrix}$$

Here P and Q are matrices involving only the $x^n y^{K-n}$ terms at float and altimetry points respectively. The solution supplies heights on the polynomial surface determined by the α's. A polynomial of order K = 12 has been successfully used to produce a set of solutions similar to Figure 2.

But is it right?

We look for a tangible characteristic of the results to test against other observations. By inspection we have found that over 2003 at least, the north-south gradient of sea surface height predicted using only Argo data in a box spanning 40° to 10°W and 30° to 50°N is constant in time at -0.02m/° equivalent to an east-west current component of 2cm/s. For comparison we have looked at data from two other sources: Topex/Poseidon satellite altimeter data combined with GRACE geoid estimates and slopes from the 1/4° OCCAM model calculated from surface geostrophic currents.

Figure 3 shows mean slope from these four sources along with envelopes at two standard deviations. There is good agreement.

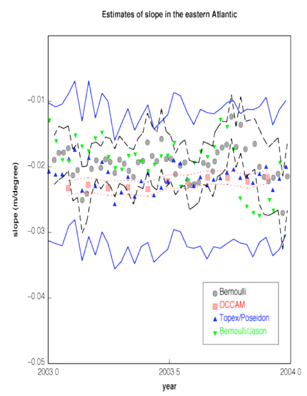

Figure 3. Slopes calculated from all points in a box covering 40-10°W and 30-50°N for Bernoulli heights (black), OCCAM model (red), Topex/Poseidon altimetry with GRACE geoid (blue) and Bernoulli/Jason combined (green)

What's next?

- Examine the assumptions behind the inverse method and how it handles errors in its input data.
- Compare this inverse method with one using the Montgomery function - this is easier to calculate and allows us to use salinity and potential temperature as conserved variables.
- Examine any differences between solutions with and without altimetry and compare to optimal interpolation methods.

Further Reading

- http://www.noc.soton.ac.uk/JRD/PROC/sga/bernoulli
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